Stocks valuation

The discounted dividends model

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v2019.08.1

Abstract

This documents defines common stocks and their characteristics. The focus is then on stocks valuation, and the discounted dividends model.

Keywords: financing, equity, stocks, dividends, valuation

Contents

Learning Objectives

By the end of this module, students should be able to:

- Define common stocks, list their characteristics
- Cite the various payments to common stocks holders
- Explain the rationale underlying the discounted dividends model
- Use the Gordon model to estimate a stock price or return
- Relate common stocks and firm valuations

1 Common stocks presentation

1.1 What is a common stock?

Definition A common stock is a security representing a share of the total equity capital of a given corporation.

This means that a common stock owner is the owner of a fraction or share of a company. Stocks owners are called *shareholders* or *stockholders*. There might be a (usually legal) difference between those two, but in the context of this document, stockholders and shareholders are identical and both words will be used with the same meaning.

Figure 1: Chicago Great Western Railway Company common stock

Example 1

Figure 1 displays an ownership certificate of 100 shares of common stock of the Chicago Great Western Railway Company.

Different kind of rights are associated or attached to common stocks ownership:

- The right to vote in shareholders meetings, thus participating in the decisions about the management of the company.
- The right to get some information about the company's performance and prospects, in order to make informed decision when voting or when deciding about one's participation in the company.
- Financial rights on the company's earnings and net worth in case of liquidation.

This document will focus on the financial rights (that is, the payments attached to common stocks ownership) and their relationship to common stocks valuation.

1.2 Common stocks payments and performance

Cash flows to the stockholders The shareholders, investors who bought some shares of stock, expect some cash flows from their investment – as any investor.

The cash flows from investing in common stocks might be:

- Dividends: periodic payments, usually in cash, depending on the company's performance (earnings).
- Capital gains: cash flow from divesting: selling a share of stock for a given price on the stock market.

Other kind of non-cash payments are possible, such as the allocation of new stocks on some special operations on the company's capital.

Note that compared to the company creditors (investors who *lended* money to the company), and especially the bond holders, the stockholders are *residual claimants*: the dividends they expect is based on the company earnings, which are what is remaining after other stakeholders have been paid: employees (wages), creditors (interests), state and local authorities (taxes), etc. Thus, the shareholders are paid *after* the other stakeholders, and with a *lower priority*: if the result is a loss (nothing is remaining), there will not be any dividend in the given year. The same holds if the company stops its activity and the assets are liquidated: the claim of the shareholders on the cash received from the liquidation has the lowest priority.

Performance indicators Investors are of course interested in their investment profitability. The ultimate measure is the investment return which we will study in more details in the next section. But the return depends on the stock price and dividends, which in turn depend on the company profitability. Thus, some performance or profitability indicators are regularly calculated and published by companies themselves, and also analysts. You can easily find those indicators on all financial information media.

Here are the most common ones:

P/E Price earnings ratio: the ratio of the stock price to the earning per share (see below).

EPS Earning per share: the (last known) net earnings of the company divided by the number of outstanding shares.

DPS Dividend per share: the (latest) dividends paid divided by the number of outstanding shares. Note that in some countries, the dividends are paid more than once per year: every quarter in the USA, for example.

Dividend yield The ratio of the DPS to the stock price.

Example 2

Figure 2 shows various indicators for the General Electric Company, as of Sept. 26, 2014:

- The EPS was 1.46. As there were roughly 10 billion shares outstanding, that means that the latest earnings were around 14.6 billion USD at that time.
- The P/E was 17.54. As the EPS was 1.46, we can calculate that this assumes an average price of

$$17.54 \times 1.46 = 25.61$$

  The opening price was 25.58 and the closing one 25.63 on that day, thus 25.61 is indeed a good estimation.
- Finally, the dividend value and yield were 0.22 / 3.43%. This means that the last (quarterly) dividend paid was 0.22. Assuming again an average price of 25.61, the estimated annual yield

was indeed

$$(0.22 \times 4)/25.61 = 0.0343 = 3.43\%$$

Figure 2: General Electric Company indicators

Source: Google Finance[1]

Stocks prices and returns As for any investment, we might calculate the return on an investment in common stocks by stating that the initial investment (cash outflow), that is, how much we paid for the stocks, should be the present value of the cash flows we got later from this investment, discounted at the return we are looking for.

From the above, we know that the cash flows are the dividends we might get over the holding period, and the sell price of the stocks once we decide to divest.

Thus, the basic equation of the relationship between a stock price, its return, and the payments it provides is:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{P_n}{(1+r)^n} \tag{1}$$

P_0 Stock price at time zero
D_1 Dividend received in the end of the first period
D_2 Dividend received in the end of the second period
D_n Dividend received in the end of period n
P_n Stock price in the end of period n
r Stock return

In the simplest case of an investment over one period of time only (e.g. a year or a quarter), we can deduce from the above that the return is the ratio of the cash flows received over the period to the stock price we paid (the investment) in the beginning of the period, minus one.

$$r = \frac{(D_1 + P_1)}{P_0} - 1 = \frac{D_1}{P_0} + \frac{P_1}{P_0} - \frac{P_0}{P_0} = \frac{D_1}{P_0} + \frac{(P_1 - P_0)}{P_0} \tag{2}$$

The notation is the same as above.

In other words, we compare the *revenues* we got in the end of the period (dividend and capital gain) to how much we paid in the beginning of the period.

Note that we can clearly see here that the return on the common stock has two components: the dividends return (or yield), D_1/P_0, and the return on the capital gain, $(P_1 - P_0)/P_0$.

[1] http://www.google.com/finance?q=NYSE%3AGE&ei=ZMYmVLiRAsKpwAOpq4DoAg

Example 3

Consider the information about General Electric Company on figure 2.

If you bought the stock at the 52 week lowest price and sold it today (closing price), what was your return? (Use the dividend yield provided and consider the holding period was one year)

We use the dividend yield provided (3.43%) and then add the return from buying the stock at 23.50 (the 52 weeks low) and selling at at the given closing price, 25.63:

$$r = 3.43\% + \frac{25.63 - 23.50}{23.50} = 12.49\%$$

Let us come back to the general case, equation 1. Its use *ex-post* is easy: we can calculate the yearly return we had after a given number of years using this equation.

Date	May 2016	Aug 2016	Nov 2016	Feb 2017	May 2017	Aug 2017	Nov 2017	Feb 2018
DPS	0.57	0.57	0.57	0.57	0.63	0.63	0.63	0.63

Table 1: Apple Inc. Dividends per Share, May 2016-Feb 2018

Source: Yahoo! Finance[2]

Example 4

You bought a share of stock of Apple Inc. (AAPL) at a price of 93.99 in the beginning of February 2016. You then received the dividends as described in table 1.

After receiving Feb. 2018 dividend, you immediately sold the share of stock on the market, at 172.43.

What was your return on this investment? We can solve equation 1 for the return for that. First, we write the equation, r being the unknown return:

$$93.99 = \frac{0.57}{(1+r)} + \frac{0.57}{(1+r)^2} + \ldots + \frac{0.63}{(1+r)^7} + \frac{(0.63+172.43)}{(1+r)^8}$$

In this expression, r is known as the internal rate of return or IRR. There is no way to calculate it explicitely so we use a spreadsheet to calculate it numerically.

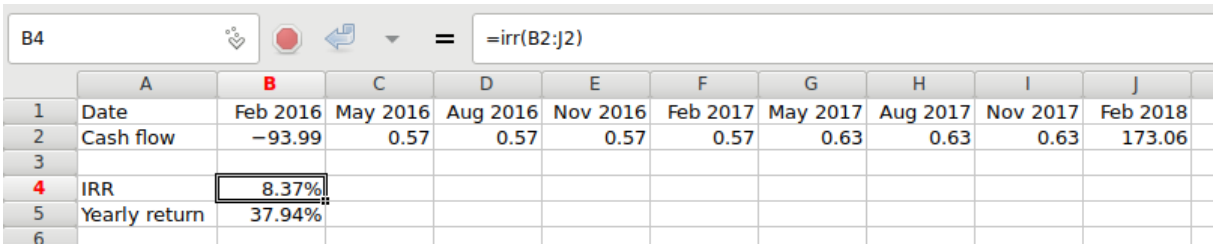
Fig. 3 shows the detailed calculation. Notice that the price you paid for the share of stock in the beginning is negative (as it was a cash outflow). The cash flow in period 8 is the sum of the dividend received, 0.63, and the sell price of the share of stock, 172.43.

Finally, the IRR function provides the result, which is 8.37% here. This is the return *per quarter*, as we received quarterly dividends in this example. Thus we have to annualize it to get a yearly return:

$$(1 + 0.0837)^4 - 1 = 0.3794 = 37.94\%$$

[2] https://finance.yahoo.com/quote/AAPL/history?period1=1362157200&period2=1519923600&interval=1wk&filter=history&frequency=1wk

Note: See for example ”Other variables” in the Time Value of Money chapter[3] for more details on the IRR calculation.



	A	B	C	D	E	F	G	H	I	J
1	Date	Feb 2016	May 2016	Aug 2016	Nov 2016	Feb 2017	May 2017	Aug 2017	Nov 2017	Feb 2018
2	Cash flow	−93.99	0.57	0.57	0.57	0.57	0.63	0.63	0.63	173.06
3										
4	IRR	8.37%								
5	Yearly return	37.94%								
6										

Figure 3: Internal Rate of Return calculation

But what if we wanted to use this model *ex-ante*, to evaluate a stock price? We could probably decide the minimum return we want on the stock, for example by observing the return on similar companies on the stock market, but how can we “guess” the future dividends, and even more difficult, the price in the end of the holding period? In some situations, some acceptable solutions to this problem exist. Read ahead.

2 The discounted dividends model

So, imagine we want to use equation 1 *ex-ante*, that is to actually get a price. A common example might be the calculation of the maximum price you are willing to pay for a stock, given the minimum return you are willing to accept. To do the calculation we would need the future cash flows, that is the dividends over the holding period, and the price at which we would sell the stock in the end. As we don’t know about the future, this seems to be nearly impossible.

2.1 Getting rid of future prices

Let us first deal with the *last* cash flow in the equation, that is, the price at which we would sell the share of stock once we decide to sell back the stock on the market (thus divesting).

It is actually far easier than it looks on first sight: if we sell the stock, someone has to buy it. The buyer will be in the same position as we are now regarding the stock, that is, the buyer will be willing to pay a price such as she gets some expected return, given some future cash flows which again are dividends, and a sell price in the end.

We assume here that the expected return is the same for you and your buyer[4].

So that means that in equation 1, we replace P_n, the sell price in the end, with the present value of the dividends from $n+1$ to another and later date, say m, and the present value of the selling price in m, P_m. Thus, equation 1 becomes:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{D_{n+1}}{(1+r)^{n+1}} + \ldots + \frac{D_m}{(1+r)^m} + \frac{P_m}{(1+r)^m}$$

It looks like we are only pushing further away in the future the problem of finding the sell price in the end. But of course you realize that the price in the end, P_m can in turn be replaced with the present value of a

[3] https://files.bagneris.net/finance/

[4] This is not a too unrealistic assumption: the risk-return relationship theory in finance tells us that the expected return for a given level of risk should be the same for all investors at a given point in time, and that these returns should be reasonably stable over time, provided the risk level does not change (the systematic risk associated with the stock does not change).

stream of dividends and a final price, which in turn can be replaced by the present value of a stream of dividends… *ad infinitum*.

Thus equation 1 becomes:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{D_{n+1}}{(1+r)^{n+1}} + \ldots = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \tag{3}$$

How does it help? It might sound crazy to replace a finite equation with an infinite one, but remember that in mathematics, some limits towards infinite are well defined. We will take advantage of this in the next section.

2.2 The Gordon-Shapiro model

Equation 3 above shows that at any given point in time, the stock price should be the present value of an infinite stream of future dividends. Of course, if we could define a relationship between the dividends, something like “the dividend at a given point in time is defined by the dividends paid before”, then the equation would probably simplify. Is it reasonable to assume such a relationship between successive dividends?

Actually, one can easily observe that most dividend-paying companies tend to practice what is known as *dividend smoothing*:

- they avoid changing the dividend per share too often,
- they try hard to never cut the dividends stream, and to never lower the dividend per share.

As a result, the stream of dividends per share is usually far smoother than the earnings per share over the same time period. That is of course perfect for our calculations: if the dividends per share exhibit a steady and slow growth, it is easy to calculate the growth from historical data. Then, given what we just explained, expecting this growth to carry on in the foreseeable future does not sound such an unreasonable assumption.

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
DPS	2.25	2.25	2.35	2.50	2.50	2.55	2.55	2.60	2.60

Table 2: Air Liquide Dividends per Share, 2009-2017

Source: Yahoo! Finance[5]

Example 5

Table 2 shows the dividends per share paid by the Air Liquide company from 2009 to 2017.

You might first notice that this time series exhibits exactly the behaviour we described above: the dividend per share changed only four times over the eight years, and experienced a slow and steady growth.

What is the average (compounded) growth per year in %?

Well, it should be g such as:

$$D_{2017} = D_{2009} \times (1+g)^{(2017-2009)}$$

Thus we get:

$$2.60 = 2.25 \times (1+g)^{8}$$

$$g = \left(\frac{2.60}{2.25}\right)^{1/8} - 1 = 1.82\%$$

The dividend per share grows on average by 1.82% per year.

So if we can calculate an average growth rate g for the dividends, we can now replace the successive dividends in equation 3, using the relation:

$$D_n = D_1 \times (1+g)^{(n-1)}$$

And equation 3 becomes:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_1 \times (1+g)}{(1+r)^2} + \ldots + \frac{D_1 \times (1+g)^n}{(1+r)^{n+1}} + \ldots = \sum_{t=1}^{\infty} \frac{D_1 \times (1+g)^{(t-1)}}{(1+r)^t}$$

The sum to infinity is a well known limit and simplifies as:

$$P_0 = \frac{D_1}{(r-g)} \tag{4}$$

The stock price depends on the next dividend per share to be paid, the dividends growth, and the expected return on the stock.

This result was published in 1956 by Myron Gordon and Eli Shapiro, and is known as the Gordon growth model, Gordon-Shapiro model or discounted dividends model[6].

[5] https://finance.yahoo.com/quote/AI.PA/history?period1=946832400&period2=1519232400&interval=div%7Csplit&filter=div&frequency=1mo

[6] https://en.wikipedia.org/w/index.php?title=Dividend_discount_model&oldid=815473168

Example 6

Taking again the data from example 5, assume that the dividends growth that we calculated will hold forever in the future (more on this assumption below). What is the maximum price you would pay for an Air Liquide share of stock if you expect a 14% per year minimum return and the company announces a 2.68 dividend per share for next year?

From equation 4, we get:

$$P_0 = \frac{2.68}{(0.14 - 0.0182)} = 22.00$$

2.3 Limits of the discounted dividends model

Of course, the discounted dividends model is just a model: it helps us understand the real world, but cannot (and is not intended to) perfectly mimic it. The assumptions behind the model should be taken seriously and do not always hold:

- some corporations have chaotic dividend streams, or even don't pay any dividends for long time periods,
- the growth rate should be *reasonable*: for example, in an economy growing by 2% per year on average, an 8% growth rate cannot be expected to be sustainable forever,
- moreover, of course, if the growth rate is greater than, or equal to the expected return on the stock, the model is nonsensical.

About the assumption of an infinite growth, it is actually less important than it seems: the effect of discounting lowers the weight of the future dividends, and assuming that the dividend is stable after a given number of years of growth makes little difference on the result of the model.

B9 = =(D_1/(r-g))*(1-((1+g)/(1+r))^t)+D_1*(1+g)^(t+1)/(r*(1+r)^t)

	A	B	C	D	E	F	G	H	I
1	D_2017	2.6							
2	D_2009	2.25							
3	2017 - 2009	8							
4	g	1.82%							
5	D_1 = D_2018	2.68							
6	r = k_E	14.00%							
7									
8	Years of growth	1	2	5	10	25	50	60	Infinite
9	Price	19.76	20.00	20.58	21.20	21.86	22.00	22.01	22.01
10									

Figure 4: Finite number of years of dividend growth

Example 7

Taking again example 6, let us calculate the price resulting from the discounted dividends model, but for a finite number of years of dividends growth. After that number of years, the dividend is assumed to stay stable.

Figure 4 shows the results of such a simulation with the Air Liquide data. You might notice that the

stock price difference between 10 years of growth and infinite growth is only:

$$\frac{(22.01 - 21.20)}{22.01} = 3.70\%$$

Of course, the higher the growth rate, the more important the difference. But as we said previously, sustainable long term growth rates are probably quite low in nowadays economies.

3 Common stocks and firm valuation

Of course, the valuation of common stocks is related to the firm valuation: if you were to buy all the outstanding shares of stocks of a given company, you would be the sole shareholder, and owner of the company. Note that, as the owner of the company, you would also be the "owner" of its liabilities.

Buying all the assets of a company, and buying all its stocks are two different operations: the assets, on the left side of the balance sheet, would be sold by the company itself. The stocks do not *belong* to the company, but are the securities underlying its equity resources. The stocks belong to the shareholders, so buying stocks means buying those from the shareholders, if they are willing to sell.

We can summarize the above paragraphs with this equation:

$$P_0 = \frac{E}{N} = \frac{V_A - D}{N} \tag{5}$$

P_0 Stock price
N Number of outstanding stocks
E Market value of equity
D Market value of debt
V_A Market value of assets, including cash at hand

The stocks represent the market value of the equity, but usually not the market value of all the assets, as equity is not the only kind of resources used by companies.

The valuation of a company is accomplished by using different methods, such as multiples, comparables and discounted cash flows, but this is a different and huge topic which is not addressed here.

Summary

- A common stock is a security representing a share of the total equity capital of a corporation.
- The cash flows expected from investing in common stock are dividends and capital gains.
- The discounted dividends model relates the stock price at a given point in time, its return, and the infinite stream of all the future dividends the stock is expected to pay.
- The Gordon and Shapiro model is a discounted dividends model in which the future dividends are expected to grow at a constant rate, forever.
- The total value of all the outstanding stocks of a company is the market value of the equity capital. It is usually different from the assets total market value, as equity capital is not the only financing of the company assets: most companies also use debt financing.

Exercises

Provide all answers with 2 decimal places, but round the final result only: remember you should never round any intermediary result.

All financial data about public companies used below was retrieved from Yahoo! Finance[7] on March 1, 2018.

1. As of March 1, 2018, Apple Inc. (AAPL) financial data includes the following: last close price was 178.39 USD, last known net income (Sept. 2017) 48.35 billion USD, total dividends paid over 2017 fiscal year 12.77 billion USD, for a total of 5.07 billion shares outstanding. Calculate the P/E, EPS, DPS and dividend yield based on this data.
2. You bought 10,000 shares of Peugeot S.A. (UG.PA) stocks on May 22, 2016 for 14.09 each. One year later, you received a 0.48 dividend, and then sold back the stocks on the market for 17.95 each. What was your return on that investment?
3. The Compagnie Générale des Établissements Michelin (ML.PA) stock price was 68.70 in the end of May 2013. A 2.50 DPS was paid in the end of May 2014 and May 2015, rising to 2.85 in May 2016 and 3.25 in May 2017. The ex-dividend closing price in the end of May 2017 was 116.40. What was the return on the stock on the May 2013-May 2017 period?
4. Using again the data from Air Liquide (AI.PA) in table 2 and from example 5:
    - What would be the expected dividend for 2018? for 2019?
    - Assume the stock price average in 2017 was 102 and 2018 dividend is the expected one you just calculated. What is the return expected from this stock on the financial market?

[7] https://finance.yahoo.com/

Exercises answers

1. As of March 1, 2018, Apple Inc. (AAPL) financial data includes the following: last close price was 178.39 USD, last known net income (Sept. 2017) 48.35 billion USD, total dividends paid over 2017 fiscal year 12.77 billion USD, for a total of 5.07 billion shares outstanding. Calculate the P/E, EPS, DPS and dividend yield based on this data.

$$\text{EPS} = \frac{48.35}{5.07} = 9.54 \text{ USD}$$

$$\text{P/E} = \frac{178.39}{9.54} = 18.71$$

$$\text{DPS} = \frac{12.77}{5.07} = 2.52 \text{ USD}$$

$$\text{Div. yield} = \frac{2.52}{178.39} = 1.41\%$$

2. You bought 10,000 shares of Peugeot S.A. (UG.PA) stocks on May 22, 2016 for 14.09 each. One year later, you received a 0.48 dividend, and then sold back the stocks on the market for 17.95 each. What was your return on that investment?

   As there is only one year of investment, with no intermediate cash flows, we can use the simple case.

   Using equation 2, we can write:

$$r = \frac{(17.95 - 14.09) + 0.48}{14.09} = 30.80\%$$

3. The Compagnie Générale des Établissements Michelin (ML.PA) stock price was 68.70 in the end of May 2013. A 2.50 DPS was paid in the end of May 2014 and May 2015, rising to 2.85 in May 2016 and 3.25 in May 2017. The ex-dividend closing price in the end of May 2017 was 116.40. What was the return on the stock on the May 2013-May 2017 period?

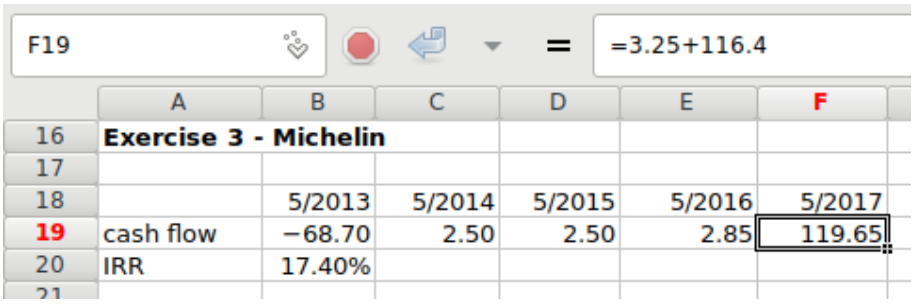


Figure 5: Michelin IRR calculation

   Figure 5 shows the calculation of the internal rate of return, based on the cash flows one would have received by investing in one Michelin stock in the end of May 2013, and selling it back on the market in May 2017 after the dividend payment. Note that the cash flow of 2017 is the dividend received in May, plus the stock price right after that. The 17.40% rate of return is calculated as in example 4.

4. Using again the data from Air Liquide (AI.PA) in table 2 and from example 5:

   - What would be the expected dividend for 2018? for 2019?

     To find the expected future dividends, we apply the growth rate found in example 5 to the last known dividend (2017):

     $$D_{2018}(\text{e}) = D_{2017} \times (1 + g) = 2.60 \times 1.0182 = 2.65$$

     $$D_{2019}(\text{e}) = D_{2017} \times (1 + g)^2 = 2.60 \times 1.0182^2 = 2.70$$

   - Assume the stock price average in 2017 was 102 and 2018 dividend is the expected one you just calculated. What is the return expected from this stock on the financial market?

     We use equation 4 with a known stock price to find the implied stock return:

     $$P_0 = \frac{2.65}{(r - 0.0182)} = 102$$

     $$r = \frac{2.65}{102} + 0.0182 = 4.42\%$$