Bonds valuation

Bonds cash flows, yield and value

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Abstract

This documents defines classical bonds and their characteristics. The focus is then on the bonds valuation, and its relationship with bonds cash flows and yield.

Keywords: financing, debt, bonds, yield, valuation

Contents

Learning Objectives

By the end of this module, students should be able to:

- Define bonds, list their characteristics and common covenants
- Explain the relationship between bonds prices and interests rates
- Calculate a bond clean or dirty price

1 Bonds definition and vocabulary

1.1 What is a bond?

Definition A bond is a *debt* security issued by governments or corporations to raise money from investors.

As the bond represents a debt owed by the bond issuer (debtor), it entails interest payments and a repayment of the principal to the bond holder (creditor).

In simple words, bonds might be seen as “small pieces” of a loan: the bond issuer, instead of borrowing from a bank or a financial institution, borrows from multiple lenders. Each of them receives a number of bonds representing the fraction of the loan they contributed to the borrower.

Bonds belong to the group of securities known as fixed income securities, because they pay interest at a rate which is determined and agreed upon in advance, thus ”fixed”[1].

Figure 1: A bond

[1] Note that some bonds use variable rates, with a market rate as a reference. The reference and the rate calculation are still fixed in advance, though, and these bonds are still considered as fixed income securities.

Example 1

Figure 1 displays an old French bond which was issued in 1903 by the Voies Ferrées Économiques ("Economical Railways") corporation.

Most details are in the attached contract, not shown on the figure, but you still can read that the nominal or face value of the bond is 100 Francs (the French currency at that time) and that the interest rate is 4% (per year).

Bonds can be issued and sold privately to investors chosen by the issuers, but many are publicly issued on financial markets, and *negotiable*: they can be freely bought and sold on the secondary market like other securities. As for any other security, the existence of the secondary market is important for a better liquidity of the bonds.

1.2 Face value and issuing prices

Face value The face value of the bond, also called *nominal*, *principal* or *par value*, is the amount the lender or creditor is supposed to have lent to the borrower or debtor. It is thus the basis for the interest calculation on the bond (remember, interests are always calculated on the basis of the unpaid balance of a loan).

Note that the face value of the bond is not necessary its issuing price, nor is it necessary its repayment value. See issuing price and repayment value below.

Many bonds characteristics, such as the price or the interest rate, are commonly expressed in percentage of the face or par value.

The total nominal amount of the loan is the number of bonds times the face value of one bond.

Example 2

The "Voies Ferrées Économiques" bond displayed on figure 1 has a face value of 100 Francs.

If there were 10,000 bonds in total, then the loan nominal value was $10,000 \times 100 = 1,000,000$ Francs.

Issuing price The issuing price is the price that the investor or subscriber has to pay for the bond when it is issued.

The issuing price might be slightly different from the face value, usually to adjust the bond to market conditions: a change in the issuing price involves a change in the bond yield, as we will see in section 2.

Example 3

Figure 2 shows the description of a bond that was issued by the Hellenic Republic (Greece) in February 2005. You can notice that the issuing price is slightly higher than the nominal or face value: it is 100.006% of the face value. Also note that, as we said before, it is expressed in percentage of the face value.

OFFERING CIRCULAR

The Hellenic Republic

Euro 5,000,000,000

3.70 per cent. Bonds due 20 July 2015

Issue Price: 100.006 per cent.

The Euro 5,000,000,000 3.70 per cent. Bonds due 20 July 2015 (the "Bonds") of the Hellenic Republic (the "Republic") will bear interest from, and including, 22 February 2005 at the rate of 3.70 per cent. per annum. Interest on the Bonds will be payable annually in arrear on 20 July of each year. The first payment of interest will be paid on 20 July 2006 in respect of the period from, and including, 22 February 2005 to, but excluding, 20 July 2006 (long first coupon). Payments of interest in respect of the Bonds to non Greek residents will be made without deduction for or on account of Greek taxes. Payments of interest in respect of the Bonds to Greek residents will be made after a deduction in respect of Greek income tax. The Republic will not pay additional amounts as may be necessary in order that the net amounts received by a Greek resident after such deduction equal the respective amount of interest which would have been receivable in respect of the Bonds in the absence of such deduction. See also "Greek Taxation" herein.

The Bonds will mature on 20 July 2015.

Figure 2: Hellenic Republic 3.70% February 2005 bond

1.3 Repayment and maturity

Maturity date The maturity date is the date at which the bond will be repaid by the issuer (debtor) to the bond holder. After that, the bond has been repaid and non longer exists.

The maturity of a bond is the total number of years from its issue date to its maturity date. As bonds are medium to long term securities, bond maturities typically are from 3 years to 30 years. Longer maturities exist (up to 100 years, such as the July 1993 Walt Disney company bonds, known as “Sleeping Beauty bonds”) but are less common.

Repayment method In the most common case, all the bonds for a given loan share the same maturity date and are repaid at the same time: the loan is repaid *in fine* (in the end). Those loans are often called “interest only loans” because until the very last day, only interest is paid by the issuer: the capital is fully repaid on the last day.

Other repayment schemes are possible, such as grouping bonds in series, and repaying each series at a different maturity date.

Repayment price The bond repayment price, the money that is returned by the issuer to the bond holder at maturity, is usually the face value of the bond. In some special cases, it might include a repayment premium and thus be greater than the face value, but of course it cannot be lower than it: you cannot repay less than the amount you borrowed. If there is a repayment premium, this is of course agreed upon from the beginning and described in the bond contract.

1.4 Coupons

The interest paid on bonds is called *coupon*.

When bonds were still pieces of paper, the bearer had to cut (in French, “couper”) the coupon and present

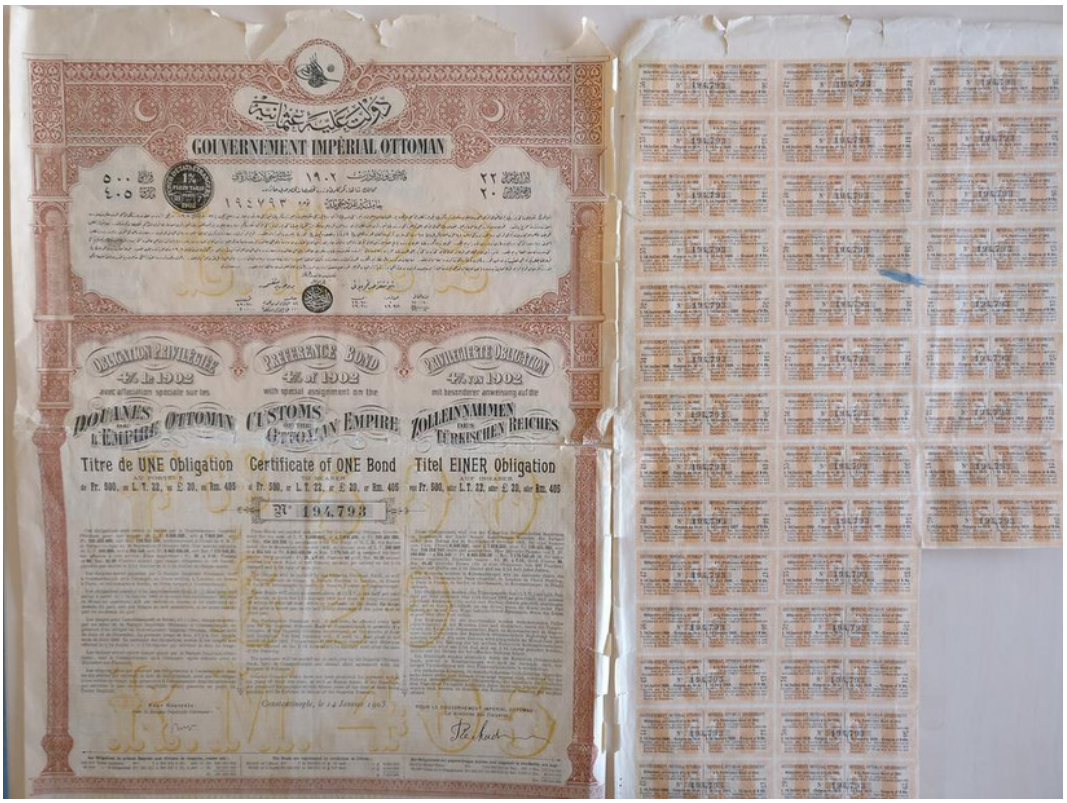


Figure 3: A bond with attached coupons (on the right)

it to a bank to receive the interest of the period, thus the name “coupon” for the interests.

Bonds coupons are paid periodically and in arrears, exactly like interest on most classical loans. Common durations of the period between two coupons payments are a quarter, half a year or a year.

The coupon rate is defined in the bond contract, as an annual periodic rate. The value of one full period coupon is given by:

$$\text{Coupon} = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{\# of Periods per Year}} \tag{1}$$

Example 4

Nowadays, coupons are paid semi-annually (twice per year) in the USA, and usually annually in France and on international bonds.

On figure 3 you can see coupons still attached to a bond (though some were paid and are missing). You can see on figure 4 that these coupons were supposed to be paid twice a year, in January and in July.

Accrued interest When bonds are sold on the secondary market, the buyer has to pay to the seller the accrued interest since the last coupon date, as on the coupon date, the full coupon is paid to the bond bearer (holder) at that time.

Figure 4: January and July 1913 coupons

The accrued interest is calculated using what is called a day count convention[2], which determines the method for counting the number of days since the last coupon payment, and the total number of days in the year. The most common convention is called Actual/Actual ICMA and uses the real number of days for both the number of days since the last coupon, and for the total number of days in the year (thus taking into account the leap years).

The bond price is called *dirty price* or full price when it includes the accrued interest, *clean price* or flat price otherwise.

Short or long coupon In order to avoid a market saturation, there is a calendar of bond issues on most markets in the world. Thus, some issuers have sometimes to wait before issuing a bond, and the maturity of a bond might not be a full number of years.

In this case, the first coupon might be paid on a period which is shorter or longer than a year and is called a short (resp. long) first coupon. Long and short first coupons are calculated using the same method as accrued interest.

Example 5

The Hellenic Republic bond pictured on figure 2 has a long first coupon, covering a period from, and including, 22 Feb. 2005 to, but excluding, 20 July 2006.

The day count convention is actual/actual. The exact or actual number of days between 22 Feb. 2005 and 20 July 2006 is 513. Thus, assuming a 1,000 € face value, the long first coupon is calculated as follows:

$$3.70\% \times 1,000 \times \frac{513}{365} = 52$$

You can check on figure 5, which shows an excerpt of the full reference document of the bond, that the long first coupon value is indeed 52 euros.

[2] See https://en.wikipedia.org/w/index.php?title=Day_count_convention&oldid=800421292

Interest:	3.70 per cent. per annum.
Interest accrual basis:	Actual/Actual (ISMA).
Interest Payment Dates:	Interest will accrue from 22 February 2005 payable annually in arrear on 20 July of each year save that the first Interest Payment Date will be 20 July 2006 (long first coupon). The amount of interest payable on the first Interest Payment Date will amount to Euro 52 for each Euro 1,000 denominated Bond. Interest Payment Dates that fall on dates that are not Business Days will be postponed to the following day that is a Business Day.

Figure 5: Long first coupon

Test yourself

Before reading further in the next section, do the small test below: find the missing words in the sentences, taken from the section you just read:

_____

A bond represents a […] owed by the bond issuer.

_____

The interest paid on bonds is called […] and is always calculated with the bonds […] value, also called the […] value.

_____

In the most common case, all bonds are repaid […]. This is called […] or […] loans.

_____

If you did not find the answers easily, search for the sentences in the section above, and make sure you understand everything. You can also use these questions in a spaced repetition software, or read again the section in a few days to check if you remember better the main ideas.

2 Bonds valuation: cash flows and yield

2.1 Bonds cash flows

When an investor buys a bond on the financial market, she expects cash flows from this investment, as for any investment. In the bonds case, the future and expected cash flows are:

- the coupons to be paid until the maturity date,
- the final repayment on the maturity date.

We can note that these cash flows are known in advance: unless the issuer fails to make the promised payments, there is no uncertainty on those, as they are defined in the bond contract from the beginning.

2.2 The price-yield relationship

As for any financial asset, at any time, the bond price on the market is the present value of the future cash flows the bond is expected to provide, discounted at the rate of return required for this bond by the agents

on the market. This rate is called the yield to maturity (YTM) and includes a risk premium called the spread, which depends on the bond default risk (see below section 2.3).

Yield to Maturity The yield to maturity, or YTM, of the bond, is the expected rate of return on the bond. It is thus the necessary discount rate to get the bond (clean) market price from its discounted future cash flows.

Thus, to get the market price of a bond, all you need is the series of the future cash flows (coupons and final repayment) until maturity, and the YTM.

Bond price The price is given by the sum of the discounted cash flows. In the simple case where the time to maturity is an integer number of periods (no broken period, no accrued interest), it is given by:

$$P_0 = \frac{C}{(1+YTM)^1} + \frac{C}{(1+YTM)^2} + \ldots + \frac{C}{(1+YTM)^n} + \frac{RP}{(1+YTM)^n}$$

$$P_0 = C \times \frac{\left(1-(1+YTM)^{-n}\right)}{YTM} + \frac{RP}{(1+YTM)^n} \tag{2}$$

P_0 Bond price
C Full period coupon
YTM Yield to maturity, per period
n number of periods until maturity
RP Repayment price (usually face value)

Equation 2 uses a common shortcut in time value of money calculations: as all the coupons are the same, the sum of the discounted coupons is the present value of a constant annuity.

Example 6

A bond will mature in 3 years from now exactly. It pays annual coupons and the coupon rate is 4.3%. The bond will be repaid at par at maturity. What is the bond price if its YTM is 4.26%?

Note that we don't know the bond face value here: it does not matter, because we will do the calculations and provide the result in *percentage* of the face value, that is, we do everything assuming the face value is 100. This is very common when dealing with bonds, and allows for easy price comparison of bonds with different face values.

The table below shows the cash flows associated with this bond. As the bond will mature in 3 years, these cash flows are 3 coupons, one in the end of each year, and the repayment in the end of year 3.

Year	1	2	3
Cash flow	4.3	4.3	104.3

Here, coupons are paid once a year, so the coupon value, using equation 1 is:

$$\frac{4.3\% \times 100}{1} = 4.3$$

In the last year (year 3), the total cash flow is given by the sum of the last coupon, 4.3, and the repayment at par (face value), 100.

We can now use the sum of the discounted cash flows, or the shortcut in equation 2 to calculate the bond price:

$$\begin{aligned} P_0 &= \frac{4.3}{(1+4.26/100)} + \frac{4.3}{(1+4.26/100)^2} + \frac{(100+4.3)}{(1+4.26/100)^3} \\ &= 4.3 \times \frac{(1-(1+4.26/100)^{-3})}{(4.26/100)} + \frac{100}{(1+4.26/100)^3} \\ &= 100.11 \end{aligned}$$

Take care: the 4.3 above is the coupon *value*, not the coupon *rate*, as we need a cash flow here. Conversely, the YTM is the discount *rate*, thus in percentage value.

Figure 6 shows the two ways to calculate the bond price: the sum of the discounted cash flows, taken one by one, or the direct method, using equation 2.

B6 = =B2*(1-(1+B4)^-B3)/B4+100/(1+B4)^B3

	A	B	C	D	E	F	G	H
1	Coupon rate	4.30%						
2	Coupon value	4.30						
3	Maturity	3	years					
4	YTM	4.26%						
5								
6	Price (direct)	100.11						
7								
8								
9	Year	1	2	3				
10	Cash flow	4.30	4.30	104.30				
11	Discounted CF	4.12	3.96	92.03				
12								
13	Price (sum of DCF)	100.11						
14								

Figure 6: Bond price calculation

Price-Yield relationship In the example above, you maybe noticed that the bond price is slightly *above* its face value, when the YTM is slightly *below* the coupon rate. This is normal and expected: the YTM is used as a discount rate in the price formula, and the higher the discount rate (the YTM), the lower the present value (the bond price).

This means that bonds prices change with their YTM changes, or more generally, with the interest rates level: as any rate on the market, the bond YTM is a market rate to which a risk premium (the spread) is added. The higher the rates, the lower the prices, the lower the rates, the higher the prices.

For bonds which are repaid at par and with a fixed coupon rate, if the YTM of the bond is higher than the coupon rate, the bond sells at a discount: its price is lower than its face value. Conversely, if the YTM is lower than the coupon rate, the bond sells at a premium: its price is higher than its face value, as in the example above.

Other calculations From equation 2 above, it is possible to do other calculations: given the bond cash flows and price, find its YTM, or find the bond coupon or repayment premium given the YTM (return), for example.

Example 7

A bond will be repaid at par in 12 years from now exactly. Its coupon rate is 2.25% (annual coupons) and its clean price 103.75. If its current YTM is 2.18%, do you think there is a repayment premium? Why? Check your answer by calculus.

To first give an answer without any calculations, we can notice that the YTM of the bond, 2.18%, is below its coupon rate, 2.25%, but close to it. Thus, the bond should sell at a premium, but with a price which is not too far from the par (100). Here the bond price is 103.75, nearly 4% above the face value. The only explanation for such a difference is the existence of a repayment premium, which, as an additional cash flow, would rise the bond price.

Check by calculation: from equation 2 above, we can find a formula for the repayment price:

$$\begin{aligned}
P_0 &= C \times \frac{\left(1-\left(1+YTM\right)^{-n}\right)}{YTM} + \frac{RP}{\left(1+YTM\right)^{n}} \\
\frac{RP}{\left(1+YTM\right)^{n}} &= P_0 - C \times \frac{\left(1-\left(1+YTM\right)^{-n}\right)}{YTM} \\
RP &= \left(1+YTM\right)^{n} \times \left[P_0 - C \times \frac{\left(1-\left(1+YTM\right)^{-n}\right)}{YTM}\right] \\
RP &= \left(1+2.18/100\right)^{12} \times \left[103.75 - 2.25 \times \frac{\left(1-\left(1+2.18/100\right)^{-12}\right)}{\left(2.18/100\right)}\right] \\
RP &= 103.91
\end{aligned}$$

The repayment price is 103.91, thus it includes a 3.91 premium in addition to the 100 face value.

Note that when the question is about the YTM, there is no explicit formula to find it, like for any other return calculation on a multi-periodic investment. You should use a financial calculator or a spreadsheet, and the *IRR* (internal rate of return) function it provides.

Example 8

A bond maturing in 7 years exactly sells at 98.54. It will be repaid at par and its coupon rate is 1.85% (annual coupons). What is its current YTM?

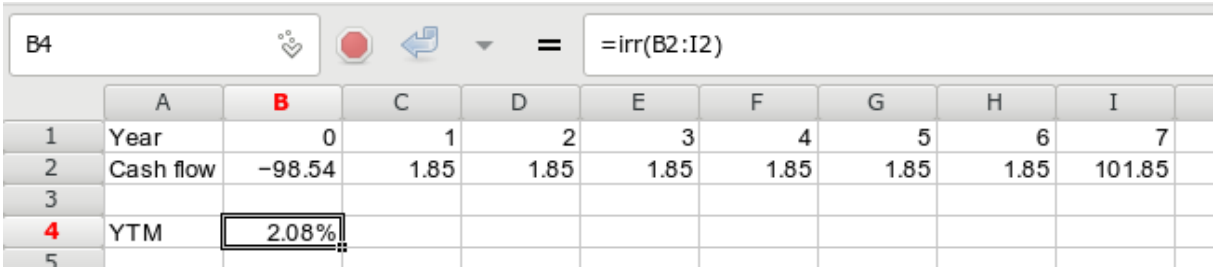
We can note before any calculation that, as the bond is repaid at par and it sells at a discount (below the par), the YTM should be greater than the 1.85% coupon rate.

From equation 2 above, the price formula for this bond is:

$$98.54 = 1.85 \times \frac{\left(1-\left(1+YTM\right)^{-7}\right)}{YTM} + \frac{100}{\left(1+YTM\right)^{7}}$$

We use the spreadsheet to solve for the YTM. The simplest way is to build a table with the bond cash flows from time 0 to maturity. The cash flow at time 0 is the price we would have to pay to buy the bond (as we would pay, it is negative), and the future cash flows are as usual the coupons and the final repayment at maturity.

Then we use the IRR() function to find the YTM from the cash flows. Figure 7 shows the final table with the cash flows and the result. Notice the YTM formula in cell B4. As expected, the YTM is greater than the coupon rate here.



	A	B	C	D	E	F	G	H	I
1	Year	0	1	2	3	4	5	6	7
2	Cash flow	−98.54	1.85	1.85	1.85	1.85	1.85	1.85	101.85
3									
4	YTM	2.08%							
5									

Figure 7: YTM calculation

2.3 Bonds risks

There are two main kinds of risks associated with bonds: default risk, and interest rates risk.

Default risk The default risk is the risk that the debtor cannot fulfill her obligations, namely, pay the interests or repay for the capital. Thus, only the creditor (bondholder, investor) bears this risk.

To inform investors about this risk, on nearly all bond markets in the world, bonds are *rated* by independent agencies: each bond receives a grade related to its default risk and the debtor capacity to meet the bond financial commitments.

The main credit rating agencies are called the “Big Three” and are Moody’s, S&P and Fitch.

Interest rates risk Both the debtor (bond issuer) and the creditor (bondholder) are exposed to the interest rate risk: it is the risk of an adverse future evolution of interest rates.

If you are the bond issuer, and the bond pays a fixed coupon (interest), then if interest rates decrease in the future, you will pay more than the market rate on your loan: you would probably prefer to repay for this bond, and issue a new one with a lower coupon rate.

If you are the bond holder, and the bond pays a fixed coupon (interest), then your position is the opposite: if interest rates increase on the market, the bond market price will decrease accordingly (the bond pays less than the market rate, it is less “interesting”), so your portfolio value will decrease.

Test yourself

The future cash flows expected from a bond are the […] and the […]. Conversely to what happens with stocks for example, these cash flows are […] in advance.

_____

To get the market price of a bond, one needs to discount its […] with its […].

_____

> The higher the YTM of a bond, the […] its market price.
>
> ---
>
> The two main kinds of risks associated with bonds are the […] risk and the […] risk.

3 Bonds special features

Bonds often have special features or covenants: special clauses in the bond contract, usually limiting or controlling what the issuer can do. The idea is to protect the bondholder (creditor) from adverse decisions from the bond issuer.

Example 9

Imagine you own 10,000 bonds issued by a given company. The bonds have a good rating and you don't worry about the company's capacity to pay the coupons and repay for the bonds at maturity.

You suddenly hear on the news that this company just issued more debt (a new loan), so that its debt ratio is now considered as dangerously high. Furthermore, the new bonds have a higher priority than yours: if the company gets in trouble, the other creditors will be repaid before you, which decreases the probability of a full repayment for you.

Of course you are unhappy with these news, and this seems rather unfair: in an unexpected way, the grade of your investment suddenly dropped.

You can figure that if what is described in the example above would be possible, people would think twice before investing in any bonds, thus reducing the possibility for companies to get that kind of financing.

To avoid this, some laws or regulations prevent companies from most extreme decisions, such as issuing new bonds with a higher priority than the previous ones. But companies can also commit themselves to avoid less extreme decisions which would also harm the bondholders position.

The most common covenants are:

- limitation of future dividend payments
- commitment to maintain the working capital to a minimum level
- restriction on further debt issuance
- call provisions

The last item, call provisions, is related to the bonds repayment. It might not be obvious, but if the bond issuer repays for some (or all of the) bonds in advance, it harms the bondholders position. In fact, if the bonds are repaid in advance, the bondholders have to find a new investment, hopefully with at least the same or a better maturity/risk/return combination. Furthermore, the bonds repaid are cancelled (they disappear), which reduces the loan liquidity and the possibilities of further trading on the remaining ones.

For these reasons, the possibilities of early repayments are regulated, and most bonds include call provisions explaining in details what is possible for the bond issuer: which fraction of the bonds can be repaid early, when, and at which price (early repayments often include a premium to compensate for the reinvestment problem).

Summary

- A bond is a *debt* security: in entails interests payments called *coupons*, and a final repayment.
- The bond price is the present value of all its future payments (coupons and repayment), discounted at the bond yield to maturity or YTM.
- The YTM depends on the bond maturity and its (default) risk. The difference between a bond YTM and a risk free rate is called the *spread*.
- The main risks associated with bonds are the default risk and the interest rate risk.
- Most bond contracts include covenants to protect the bondholders. The most common one is the call provision and details the possibilities of early repayment of the bond.

Exercises

Provide all answers with 2 decimal places, but round the final result only: remember you should never round any intermediary result.

1. A bond paying a 5.2% coupon annually has a 12 years maturity and will be repaid at par. What is its current market price if its YTM is 5.08%?
2. What is the price of a pure discount (i.e. zero coupon) bond which will be repaid at par in 8 years if its YTM is 3.45%?
3. A bond market price is 100.87. It will mature in 5 years, its coupon rate is 4.1% and its YTM 4.31%.
    - Without any calculation, tell if you think the bond is repaid at par and why
    - Calculate the bond repayment price
4. What is the coupon rate of a bond which current market price is 101.65 if its YTM is 4.72% and it will be repaid at par in 6 years from now?

Exercises answers

1. A bond paying a 5.2% coupon annually has a 12 years maturity and will be repaid at par. What is its current market price if its YTM is 5.08%?

   Before any calculation, we can notice that the bond should sell at a premium, as its YTM is lower than its coupon rate and the bond is repaid at par (see “Price-Yield relationship” in section 2).

   Then, from equation 2, we get:

   $$5.2 \times \frac{\left(1 - \left(1 + 5.08/100\right)^{-12}\right)}{\left(5.08/100\right)} + \frac{100}{\left(1 + 5.08/100\right)^{12}} = 101.06$$

2. What is the price of a pure discount (i.e. zero coupon) bond which will be repaid at par in 8 years if its YTM is 3.45%?

   From equation 2, and as the coupon is 0, we get:

   $$\frac{100}{\left(1 + 3.45/100\right)^{8}} = 76.24$$

3. A bond market price is 100.87. It will mature in 5 years, its coupon rate is 4.1% and its YTM 4.31%.

   - Without any calculation, tell if you think the bond is repaid at par and why

     As the bond YTM is greater that its coupon rate, if the bond were repaid at par it would sell at a discount. But it actually sells at a premium, which shows that it is repaid at a price higher than the par.

   - Calculate the bond repayment price

     We use the same method as in example 7:

     $$\left(100.87 - 4.1 \times \frac{\left(1 - \left(1 + 4.31/100\right)^{-5}\right)}{\left(4.31/100\right)}\right) \times \left(1 + 4.31/100\right)^{5} = 102.22$$

     We notice that the repayment price is indeed above the par: there is a repayment premium.

4. What is the coupon rate of a bond which current market price is 101.65 if its YTM is 4.72% and it will be repaid at par in 6 years from now?

   As the bond sells at a premium, we expect the coupon rate to be greater than the YTM.

   From equation 2, we isolate the coupon value, and we get:

   $$\left(101.65 - \frac{100}{\left(1 + 4.72/100\right)^{6}}\right) \times \frac{\left(4.72/100\right)}{\left(1 - \left(1 + 4.72/100\right)^{-6}\right)} = 5.04$$
