Fundamentals of risk and return in finance

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Abstract

This documents introduces the risk-return relationship in finance. It then explains how to use this relationship to estimate the expected return on common stocks and other assets.

Keywords: risk, return, diversification, CAPM

Contents

Learning Objectives

By the end of this module, students should be able to:

- Define the concept of risk in finance
- Describe the risk-return relationship
- Understand the origin of the mean-variance framework
- Cite different risk measures
- Explain the diversification effect and its consequences on risk measurement in finance
- Use the CAPM to estimate the expected return on an asset

1 The concept of risk in finance

1.1 The concept of risk

Everyone knows more or less intuitively what the simple word "risk" means, but it might relate to different underlying definitions, depending on whom and when you ask.

Example 1

Even if we deliberately stay in the field of finance, it is easy to find different examples of risky situations:

- if you lend money to someone, you might not be repaid, or repaid later than expected,
- if you buy a share of stock, you might no receive the dividends you expected, or the future stock price might not grow as much as you wanted,
- if you invest in real estate property, you might have trouble to resell your property in the future, depending on the market state.

You might have noticed that the examples above have two points in common:

- they are all related to the uncertainty about the future: we do not know for sure what will happen, we have expectations about some future events and the realizations might be different,
- all of them are negatively connoted: it is as if in the end, you get less money (or return) than you expected from your investment.

About the last point, it actually does not have to be so: the concept of risk in many languages is indeed related to the uncertainty about the future, but is not necessarily negative. The future might also turn positive, as because of the uncertainty, you might as well get a payment more important than expected, for example.

In finance, the concept of risk is generally associated to the concept of uncertainty. Thus, the risk is measured using tools which deal with uncertainty, like statistics and probabilities.

1.2 Risk profiles

It is important to note that people (and investors are people) might have different profiles regarding risk as uncertainty about the future, or even can change their risk profile depending on their age, what is at stake, etc.

We define here three different risk profiles, and explain why it is generally assumed in finance that investors belong to the profile of *risk averse* people.

Risk seekers Some people take risk for the sake of it: it seems they seek the reward of the sudden adrenalin rise this provides. People playing in casinos usually play for the sake of playing for example. People uploading videos on the web where they put there life at risk by climbing skyscrappers or high cranes or antennas probably belong to the same category.

Risk neutral Risk neutral people take decision by considering only one parameter, the expected future outcome of the event. The risk by itself does not interfere in the decision. This is better explained with a simple example.

Example 2

You play heads or tails with a friend. The coin is not rigged, and each of you plays in turn. When the coin shows heads, you pay a given amount A to your friend, when it is tails, he pays A to you. Furthermore, you agree to play 3 times only: as the number is odd, there will be a clear winner.

Clearly, the expected value of the game result is 0 for each of you, as you have equal probabilities of receiving or paying A at each turn:

$$(50\% \times A) + (50\% \times (-A)) = 0$$

Does it matter to you if A is \$100,000 rather than \$1? If the answer is yes, then you are not risk neutral.

Risk averse Risk averse people take the risk into consideration when taking decisions, and will apply a simple rule: the higher the risk, the higher the expected outcome should be. We do not make any other assumption about the risk-outcome relationship, and especially we do not assume that it is linear. Note that this does not mean that risk averse people never take any risk: it means that they only take risk when the expected outcome is big enough – and each of them has his definition of “big enough”.

Many models in finance assume that the agents on the financial markets are risk averse: they will invest in a risky security only if the outcome they expect from their investment is “big enough”. The rest of this document will adopt the same assumption. We develop this idea in the following sections, and especially develop this idea of “big enough” outcome.

1.3 The risk in finance as uncertainty about expected return

We have enough material now to pause and define what the definition of the risk in finance will be in the following. We observed that the risk is often related to uncertainty about future outcomes, and more specifically to the difference between the future outcome and what was expected. We noticed that the difference might be positive: the future outcome might be better than expected.

In finance, and especially investment matters, the “outcome” of an operation is generally measured as its return, and we will stick to this. Thus the expected outcome will be the expected return on a given investment, and the uncertainty will be related to the set of the possible future returns.

So we define the risk in finance as **the uncertainty about future expected returns** on some investment.

1.4 The risk-return relationship in finance

Given the above definition of risk and the assumption that all agents (investors) are risk averse, we can discuss the relationship between risk and return in finance a bit further.

The logical consequence of the previous statements is that the greater the uncertainty about an investment, the greater the expected return.

We will make now an additional assumption: there are some investments which bear no risk at all (that is, no uncertainty about their future return), and these investment do have an (admittedly small) expected return.

> **Example 3**
>
> It is generally considered that there is practically no risk in investing in the debt of economically and politically stable countries: the risk that they do not face their obligations to pay the interests on time and to repay the debt at maturity is extremely low.

The return on investments without any uncertainty is called the *risk-free rate* and will be noted r_f.

Of course, any risky investment should have an expected return which is higher than the risk-free rate r_f, because of the risk aversion of the agents. Thus, any expected return is the sum of the risk-free rate r_f and some additional return depending on the uncertainty of the investment. This additional return is called the *risk premium* on the investment and depends (only) on the uncertainty on the future returns on the investment.

We now have a starting point, a very basic relationship between risk and return in finance:

$$E(r_i) = r_f + f(u_i) \tag{1}$$

$E(r_i)$ Expected return on investment i
r_f Risk-free rate
u_i Uncertainty on the future returns of investment i
$f(u_i)$ Risk premium as a function of the uncertainty on the future returns of investment i

In the following sections, we will focus on how to calibrate (associate numbers and values with) such a model, and we start with the problem of measuring the uncertainty on the future returns below.

2 Risk measurement, portfolios and financial market

2.1 Risk measurement

As we defined risk in finance as the uncertainty about future returns, our risk measurement should provide some kind of “size” of the uncertainty, the extent to which the returns can diverge from their expected value. The wider the range of the possible future outcomes, the more they can diverge from what we expect.

In addition, we would like this measure to be standardized, in order to be able to compare different assets or investments.

Finally, we should remember that we want to use this measure in a function which will translate it in a risk premium, that is, the additional expected return needed for this risk to be accepted. This is the f function in equation 1, which takes u_i, the risk measure, as a parameter.

Example 4

Take again the game in example 2. If the stake, the amount A the loser has to pay to the winner at each turn is $1, then the game is intuitively far less risky than if A = 100,000.

One simple (and not very robust) way to formalize this is to calculate the range of the possible outcomes. If there are 3 turns in the game as in example 2, the maximum loss is $-3 \times A$ and the maximum gain is $+3 \times A$. Thus in the risky case, the total range is $600,000 (from -300,000 to +300,000), and in the less risky one, it is only $6 (from -3 to +3).

We thus want a measure of *scope*, such as the range. We certainly do not need to create anything from scratch here. The problems related to uncertainty have been studied carefully for centuries, and robust tools exist: we just have to use them properly.

Thus, the starting point for our risk measurement will be the *standard deviation* of the future returns. It is a well known, robust and simple to use measure, and its calculation is available in nearly any electronic calculator or spreadsheet.

2.2 Diversification and risk rewarding

Don't put all your eggs in a single basket.

Interestingly, the story of the measurement of the uncertainty of future returns does not stop with the standard deviation. Following the advice of common wisdom, most investors do not invest all of their money in the same asset. Using financial vocabulary, we would say that instead, they have more than one line in their portfolio.

Stock	Total	Average	Std. Deviation
Walmart Inc.	39.48	0.77	2.36
Exxon Mobil Corporation	-1.91	-0.04	1.43

Table 1: Walmart Inc. and Exxon Mobil Corp. weekly % returns over 2017 - Summary.

Data source: Yahoo! Finance[1]

The observation of the behaviour of portfolio returns, and especially the uncertainty on future portfolio returns, leads to interesting discoveries, which in turn will allow us to refine our risk measurement.

Let us start with an example.

Example 5

I downloaded the weekly adjusted closing price for the year 2017, for the common stocks of Walmart Inc. on the one side, and for Exxon Mobil Corporation on the other side (the two companies

[1] https://finance.yahoo.com/quote/

were chosen at random in two different sectors).

From the prices, weekly returns were calculated in a quick and very simple way[a]. The summary of the two series of returns are provided in table 1: the mean and the standard deviation of the returns over the year are provided for both stocks.

Note that we do not focus on the raw performance here: the time span of one year is too short to assess correctly the profitability of each company.

Imagine now that we want to invest money with the minimum possible level of risk, and have no other choices than these 2 stocks available. You might be surprised to realise that it is indeed possible to achieve a positive return over the year, with a risk (standard deviation) lower than the one on Exxon Mobil Corp., simply by combining the two stocks in a portfolio.

This maybe surprising result is related to the fact that the returns of the two stocks I chose are not *perfectly correlated*: they do not evolve in the same way. Thus, the movements of one might partially offset the movements of the other one, resulting in a smaller range of possible values for the portfolio, that is, a lower risk. This is clearly visible on figure 1: the blue dotted line is for Walmart returns: it clearly exhibits more spikes, either up or down, than the cyan one, which is for Exxon Mobil returns: indeed, the standard deviation of returns is lower for Exxon. We also see easily that their relative movements are sometimes opposite, like in early November.

I created a portfolio, invested at 30% in Walmart and 70% in Exxon. Its returns are plotted with the red line on figure 1. It is easy to see that the red line is smoother than the blue and cyan ones, and that the portfolio return is “somewhere between” the returns of the two stocks. It is especially striking in February, early June and end of October. Indeed, the performance of the portfolio is reported in table 2: its standard deviation is lower than the ones of the two stocks, and its performance is positive. That is, it has a both a lower risk and a higher performance than the less risky stock, Exxon Mobil.

[a] Continuous weekly returns, dividends ignored.

The effect which is illustrated in the above example is known as the *diversification effect*. Because the returns of different stocks on the stockmarket are not perfectly correlated (the stock prices do not “move perfectly together”), if we associate the stocks in a portfolio, some of the variations will offset each other. In other words, we might get rid of part of the risk only by combining different stocks in the porfolio, instead of investing all our money in only one line.

This effect is actually somewhat limited: we cannot remove all the risk by adding more line of stocks in a portfolio, whatever the care we take in choosing those. There will be a limit, a level of risk below which we cannot go, whatever the number of lines in the portfolio. Can you imagine why?

Portfolio	Total	Average	Std. Deviation
30% Walmart + 70% Exxon	10.51	0.21	1.14

Table 2: Portfolio performance.

Data source: Yahoo! Finance[2]

Well of course we cannot totally get rid of the uncertainty related to the future. But what exactly does limit the diversification effect when we add more lines in a portfolio? Interestingly enough, it is the fact that,

[2] https://finance.yahoo.com/quote/

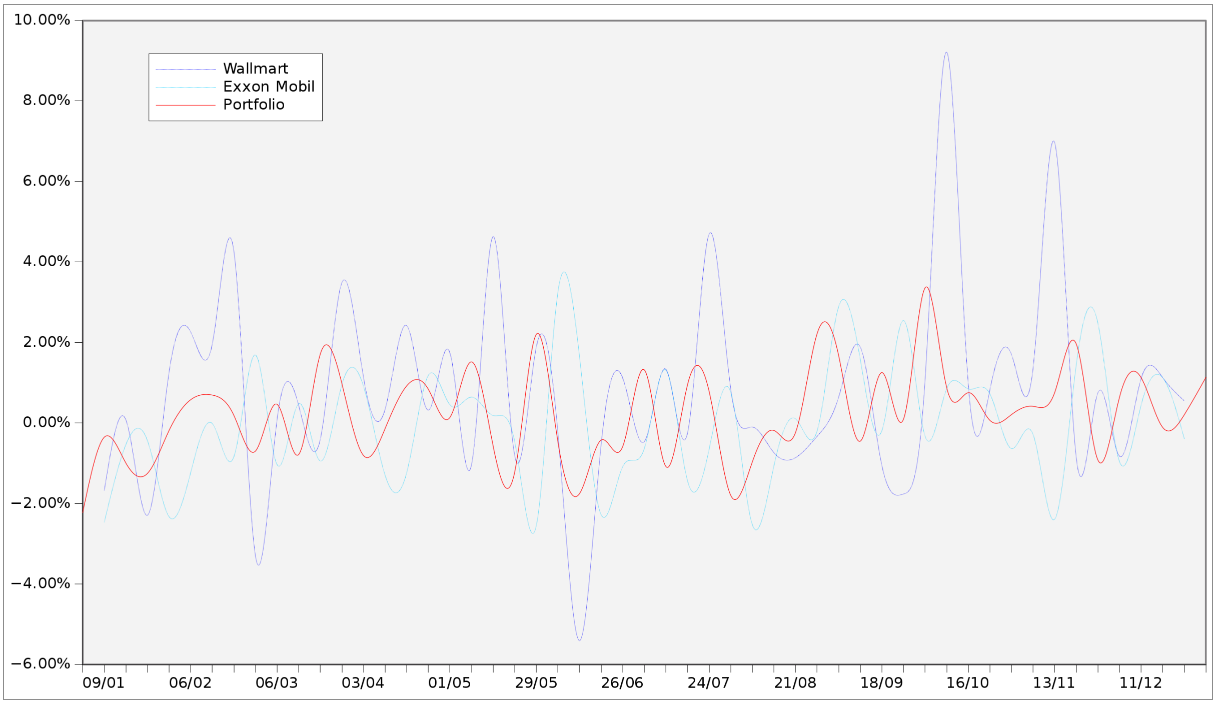

Figure 1: Walmart Inc. and Exxon Mobil Corp. weekly returns over 2017.

though not perfectly correlated, the stocks returns are *somewhat* correlated, though. The fact is, all the companies which issued these stocks compete in the same economy – which, by the way, is more and more “global”. That means that some uncertainty factors will affect all the companies, even if not to the same extent. We can think of factors such as the energy price, the interest rates level: all companies use energy and financial resources, thus would be affected by a change in those variables, but of course not to the same extent.

Example 6

A rise in the oil price would affect an airline far more than a private school. The fuel price is one major cost for the airline, but for the private school, it is barely and indirect and minor cost, probably related to the lightning and heating of the classrooms.

Thus, we can think of the risk which can be suppressed through efficient diversification as the part of risk which is specific to a company, and does not affect others. Conversely, the part of the risk which cannot be suppressed is the one which is common to all companies in the economy, even if it does not affect each of those to the same extent.

Example 7

Imagine two companies producing and selling microprocessors, such as Intel and AMD. If security researchers discover a flaw in the design of the processors of one of the companies, this will very likely affect its sales, and profit to the other one. This risk is *specific* to the affected company: it touches it, but not the other one.

If you are an investor, and have stocks of both companies in your portfolio, then you are barely affected by the flaw: the negative effect on the affected company stocks will likely be compensated by

> the positive effect on its competitor ones. The specific risk is suppressed by the diversification.
>
> But if there is a sudden rise in the price of the raw material necessary to produce the microprocessors, then both companies will be affected: this risk is not specific to one of them.

The risk which is diversifiable is called *idiosyncratic* or *specific* risk. The one which is not is the *systematic* risk[3].

One important consequence of the diversification effect is that the specific risk will not be rewarded on an efficient market: there is no reason to pay a premium for some risk that you can suppress for free, simply by diversifying your portfolio. Thus we can refine the statement of the previous section: as the investors are risk averse, any given return is the sum of the risk free rate, and a premium which is a function of the systematic risk. Equation 1 thus becomes:

$$E(r_i) = r_f + f(s_i) \tag{2}$$

s_i is the systematic risk of investement or asset i.

2.3 Measuring the systematic risk

Let us now find a way to measure the systematic risk, the one which is rewarded. The standard deviation of returns we used before measures the total risk, so it is no longer suitable for our purpose.

Remember we said that the systematic risk exists because all the stocks are somewhat correlated: all the companies "play" in the same economy. By the way, you probably noticed that financial news tell us everyday how a given financial market fared as a whole on this day, usually through a market index performance. This information is useful, because, as the companies are all affected by the same global economic variables (though, again, not to the same extent), they tend to "move together": in a rising market, there are not many stocks which exhibit negative returns.

So one way to measure the systematic risk would be to estimate to which extent one stock returns fluctuations are related to the fluctations of the returns of all the other ones on the market. We measure "all the other stocks returns" as the *market returns*, that is, the global return of all the stocks in the market. Then, the extent to which a given stock returns "move with" the market returns is measured by the *covariance* of the stock and the market returns.

So far, so good, but the covariance is not a very convenient measure: it is very difficult to appreciate it beyond its sign. If the covariance is positive, then the returns tend to "move together", if it is negative, they "move in the opposite directions": the stock returns usually drops when the market returns rise. The reason why it is difficult to tell more is because the covariance is not a relative but an absolute value, and is calculated by multiplying returns, which does not mean much (at least to me).

Data source: Yahoo! Finance[4]

> **Example 8**
>
> Table 3 shows a summary of the weekly returns of Intel Corporation and the NASDAQ Composite

[3] Note that this is different from the *systemic* risk, the risk that a whole system collapses due to cascading failures.
[4] https://finance.yahoo.com/quote/

Returns	Total	Average	Variance
NASDAQ Composite	22.34	0.44	0.000131
Intel Corporation	26.33	0.52	0.000626
Covariance			0.000118

Table 3: NASDAQ Composite index and Intel Corp. weekly returns over 2017 - Summary.

index over 2017 (Intel Corporation stocks are indeed listed on the NASDAQ).

The full example is available online[a] as a Google Spreadsheet[b].

In addition, the table shows on the last line the covariance of Intel returns and the NASDAQ index (thus the market) return. As we said before, it is very difficult to tell much about this covariance, except that it is positive.

[a] Note that you cannot edit directly the spreadsheet online: you should download it and work on your local version.
[b] https://docs.google.com/spreadsheets/d/1sj9GMB22rtDS6Y7loQaoksFR-eRrdAAHN7_KEqG87rk/edit?usp=sharing

One solution to this problem is to make the covariance relative, so that it is easier to interpret. But relative to what? Well, we said above that we care about the systematic risk, and that it will be measured by the extent to which a stock returns "move with" the market returns. We thus choose to calculate the covariance relative to the market risk as a whole (its variance). That is, we express the systematic risk in *units of market risk*.

The final systematic risk measure is called the beta, and is defined by:

$$\beta_i = \frac{COV(r_i, r_M)}{\sigma^2_{r_M}} \tag{3}$$

β_i the beta of company i
r_i company i returns
r_M market returns
$COV()$ the sample covariance
σ^2 the sample variance (squared standard deviation)

Example 9

Taking again the data of example 8, the Intel beta calculation would be:

$$\beta = \frac{0.000118}{0.000131} = 0.8982$$

How is this more meaningful than the mere covariance? It tells use that the systematic risk of Intel is roughly 0.9 units of market risk, that is, slightly below the risk of the economy as a whole.

Note again that the calculations done here are simple and naïve, that I used weekly returns over one year only, and that the resulting beta is by no means supposed to be a correct estimation. The data and results here are intended to be used as an example only.

We can observe that the beta of the whole financial market is obviously one, as it is the reference. If the beta of an asset is lower than one, then this asset's systematic risk is smaller than the market risk. Conversely, if the beta of the asset is greater than one, then this asset's systematic risk is greater than the market risk. More precisely, the asset's systematic risk is beta times the market risk.

Note that the beta (as the covariance), might be negative, in which case the asset might be riskier or less risky than the market, but "moves in an opposite way": it will generally drop when the market rises and conversely.

2.4 The capital asset pricing model (CAPM)

A word of warning: in this section, I will deliberately skip many concepts, developments, and reasoning that lead to the CAPM, as well as any proof of it. The purpose here is only to introduce the model and the way to use it. Thus, it is recommended that, once you grasp the basic concepts, you turn to a classical textbook to get the complete picture.

We can now rewrite equation 2, as we decided to measure the systematic risk with the beta:

$$E(r_i) = r_f + f(\beta_i) \tag{4}$$

That is, the expected return on an asset (or a security) is the sum of the risk free rate and a premium which is a function of the beta of the asset.

What does this function look like? Fortunately, it could not be simpler: it states that the appropriate risk premium $f(\beta_i)$ for a given asset is its beta, times the market risk premium. That sounds logical: a beta of 1.5 tells us that the systematic risk of an asset is 1.5 times the market risk, assuming that the asset risk premium should then be 1.5 times the market risk premium is reasonable.

If we note the expected market return as $E(r_M)$, then the market risk premium is $E(r_M) - r_f$, that is, the part of the market return in excess of the risk free rate.

Using this notation and the function *f* definition above, we can write the final version of equation 4:

$$E(r_i) = r_f + \beta_i \times [E(r_M) - r_f] \tag{5}$$

This final version of the equation is known as the Capital Asset Pricing Model, or CAPM for short. It is due to the works of Lintner and Sharpe in the mid-sixties[5].

This formula is important enough for us to spend a few more lines on it.

Let us repeat the variable definitions to insist on the elegant simplicity of the model. The expected return on any asset is simply the result of the sum of a minimum basis, called the risk free rate, and a risk premium suitable for this asset. That is:

$$E(r_i) = r_f + \beta_i \times [E(r_M) - r_f]$$

In the equation above, the red part is the risk premium for the asset *i*. We can rewrite the equation this way:

[5]Interestingly enough, they did not work together but developed the idea independently.

$$E(r_i) - r_f = \beta_i \times [E(r_M) - r_f]$$

Which means that the risk premium for a given asset *i* is this asset's systematic risk coefficient, called the beta β_i, multiplied by the overall market risk premium:

$$E(r_i) - r_f = \beta_i \times [E(r_M) - r_f]$$

The blue part is the market risk premium, sometimes called the risk premium for short. But beware: the expected return on *any* asset is the risk free rate plus a risk premium. Better tell explicitely which asset's risk premium we are talking about.

Example 10

We estimated the beta of the ABC Company's stocks to 1.26. The risk free rate is 1.94% and the expected market return 14.05% this year. What is the market risk premium? The ABC Company's risk premium? The expected return on ABC's stocks? The market risk premium is the difference between the market expected return, and the basis return (the risk free rate):

$$E(r_M) - r_f = 14.05\% - 1.94\% = 12.11\%$$

The ABC Company's risk premium is the market risk premium multiplied by the company's beta:

$$\beta_{ABC} \times [E(r_M) - r_f] = 1.26 \times 12.11\% = 15.26\%$$

Finally, the expected return on ABC's stock is the sum of the risk free rate and ABC's risk premium:

$$E(r_{ABC}) = 1.94\% + 15.26\% = 17.20\%$$

3 Financial risk and default risk

3.1 The components of risk and the unlevered beta

Let us now think again about the risk in finance, and more precisely, about the fundamental components of the (systematic) risk for a given company.

One useful way to split the risk in components is based on the following question: why would two different companies doing the same thing have different levels of risk?

What we mean by "doing the same thing" is that they have the same kind of activities, invest in the same sectors. If the question above did not sound meaningless to you, you then implicitely assumed that part of the risk is linked to the nature of this activity. You are right: some activities (or sectors, or industries) are obviously riskier than others, and market data confirms this. The reasons for this might be the stability of consumer demand, the structure of the market, the price volatility of the main inputs or raw materials, the likelihood of a disruption in the industrial process, etc.

So, why would to companies in the same industry have different levels of risk? One important reason is the *capital structure* of the companies, the balance between debt and equity capital in their financing.

The interest payments on debt do no depend on the level of sales of the company: whatever the earnings before interest, the interests should be paid. Thus, companies with more debt will experience a higher volatility of earnings (and returns). In addition, the higher the ratio of debt to equity capital, the higher the probability of failure (bankruptcy) for the company. These two factors (volatility of earnings and probability of failure) combine to make companies with high debt riskier than similar companies with low debt.

Based on these observations, we decide to split the risk in two components:

- the activity risk, also called the industry risk or the assets risk – as the company realizes its activity by operating its assets.
- the financial risk[6], related to the company's debt to equity ratio.

Following this, we now define the *unlevered beta* or beta of assets as the beta coefficient which only measures the activity risk. In other words, this is the beta a company would have if it had no debt in its financing, only equity capital.

A useful relationship links the unlevered beta (noted β_{UL}) and the "global" beta of the company, that is, the beta of its equity:

$$\beta_{UL} = \frac{\beta_L}{(1 + (D/E) \times (1 - \tau))} \tag{6}$$

β_{UL} the unlevered beta of the company
β_L the levered beta of the equity of the company (standard beta)
D the company debt market value
E the company equity capital market value
τ the marginal corporate tax rate for the company

Note that the ratio D/E is known as the financial *leverage* of the company.

The equation shows that the company's equity beta only depends on two "factors": the risk of its activity, represented by the unlevered beta, and the financial risk, represented by the leverage, the D/E ratio. This is what we expected from this section introduction. Furthermore, the unlevered beta is logically lower than the equity beta, because we divide the latter by a factor which is greater than one, as the leverage, as well as the $(1 - \tau)$ terms cannot be negative.

To be rigorous, it should be added that we assume in equation 6 above that the company's debt has a zero beta, that is, there is no market risk on the debt – which does not mean that there is no risk in the company's debt, see next section.

Example 11

Alpha SC, Inc. operates in the semiconductor industry, which unlevered beta is estimated to 1.16 in early 2018. Assume that Alpha SC's equity market value is \$230 million, and its debt market value is \$82 million. Its marginal tax rate is 24%. What is your estimation for Alpha SC's beta (its equity beta)? Using equation 6 above, we get:

$$0.96 = \frac{\beta_L}{(1 + (82/230) \times (1 - 0.24))}$$

[6] Not to be confused with the more general concept of the risk in finance

Thus:

$$\beta_L = 0.96 \times (1 + (82/230) \times (1 - 0.24)) = 1.2201$$

You can notice in the example above that, as expected, the beta of the company is greater than its unlevered beta: the presence of debt adds risk for the equity holders.

3.2 The default risk and the expected return on debt

As we said previously, it is often assumed that a given company's debt beta is zero: the risk on the debt is not correlated to the market risk.

One of the reasons that makes this assumption reasonable is that the most important risk on any given debt is probably the *default risk*, the risk that the debtor does not fulfill its commitments (paying interests and repaying the debt on due time). The fact that a given debtor might default is rather specific, and thus not related to the systematic and market risks.

How is this risk managed? Think about what the professional money lenders (the banks) do to manage and mitigate the credit risk: they take the decision to lend or not, and charge interests depending on *information* about the debtor. This information is often bought from external professional services.

The same happens with the corporate (or even governments) debt on financial markets: some professional services, called *credit rating agencies*, provide information about debtors and their debts, aggregated as a global grade known as the rating. The rating in turns determines the risk premium on the debt, known as the **spread**.

For obvious reasons, the credit rating agencies are supposed to be totally independent from other actors in finance. The most prominent credit rating agencies are probably S&P, Moody's and Fitch.

Finally, as for any other asset or security, the expected return on any given debt is the sum of the risk free rate and a risk premium. In the debt's case, the risk premium is called the spread, and is linked to the debt's rating.

Summary

- One common definition of the risk in finance is the uncertainty about expected (future) returns on some investment.
- Assuming that agents are risk averse in the economy, the basic relationship between risk and return states that the higher the risk, the higher the risk premium which must be included in the expected return.
- An investment bearing no risk (uncertainty) about its future returns is assumed to have a minimum return called the risk free rate.
- Because the returns on different stocks are not perfectly correlated, holding different stocks in a portfolio allows to get rid of stocks-specific risk. This is known as the diversification effect.
- The risk remaining after diversification is called the systematic risk. The risk premium on assets depends on the systematic risk only, as the specific risk might be cancelled by diversification.
- The systematic risk is commonly measured in units of "market risk", that is, the global risk of the economy. The resulting risk coefficient is called the *beta*.

- The model associating the expected return on an asset and this asset's systematic risk measured by its beta is known as the Capital Asset Pricing Model (CAPM):

$$E(r_i) = r_f + \beta_i \times [E(r_M) - r_f]$$

- $E(r_M) - r_f$ is known as the market risk premium. The product of the asset's beta by the market risk premium yields the asset's risk premium.
- The systematic risk is mainly related to two components: the activity risk and the financial risk. The latter depends on the company *leverage*, the ratio of its debt market value to its equity market value.
- The beta a company would have if it had no debt, that is, if there were only the activity risk, is called the *unlevered beta*. It is related to the assets beta, the company leverage and its marginal tax rate:

$$\beta_{UL} = \frac{\beta_L}{(1 + (D/E) \times (1 - \tau))}$$

- The main risk on debt is the *default risk*, which is the risk that the debtor does not fulfill its commitments.
- Credit rating agencies grade the corporations and governments debts with a *rating* which in turn determines the *spread* on the debt, which is the basis of its risk premium.

Exercises

Provide all answers with 2 decimal places, except the betas which should have 4 decimal places. Remember to round the final result only: you should never round any intermediary result.

All market estimates such as market risk premium, risk free rate are from Prof. Damodaran website[7]. All market data is from Yahoo Finance[8].

1. According to Yahoo finance in early April 2018, Amazon.com, Inc. beta is 1.71. Estimates for the market risk premium and the risk free rate at that time were 5.08% and 2.41% respectively. What Amazon's risk premium? What is the expected return on Amazon's stocks? What is the expected return on the market?

2. Assume that the expected return on Apple Inc. stocks is currently 7.89%. Yahoo Finance estimates that its stocks beta is 1.08. If the risk free rate is 2.4%, what is the implied market risk premium?

3. Late in 2014, a privately held company's CFO estimates that its shareholder expect a 12.15% return on their shares of stock currently. If the risk free rate was 2.17% at that time, and the market risk premium 5.78%, what is the beta estimate that the CFO used to get the expected return?

4. The Space-Y company is listed on a “small caps” stock market and its stock closed at $143.58 yesterday. There is 1.2 million shares of stocks outstanding, and their beta is estimated to 1.82. The company has no debt so far, but it is considering a $50 millions bonds issue for investment purposes. If the marginal tax rate of Space-Y is 38%, what would happen to its beta?

5. In 2011, Nebula, Inc. is a privately held company. Its CFO whishes to get an estimate of the expected return on its stocks before going public, by using the CAPM. In order to estimate its beta, it found a competitor (thus a company with a similar activity) which beta is 1.32. The competitor has a 38% leverage, but Nebula's leverage is estimated to 0.18 only. Assume that both companies have a marginal tax rate of 30% and use 1.88% and 6% as the risk free rate and the market risk premium, respectively. What is your estimation for the expected return on Nebula's stocks?

[7] http://people.stern.nyu.edu/adamodar/New_Home_Page/datacurrent.html
[8] https://finance.yahoo.com/

Exercises answers

1. According to Yahoo finance in early April 2018, Amazon.com, Inc. beta is 1.71. Estimates for the market risk premium and the risk free rate at that time were 5.08% and 2.41% respectively. What Amazon's risk premium? What is the expected return on Amazon's stocks? What is the expected return on the market?

   The CAPM teaches us that the risk premium on Amazon is Amazon's beta times the market risk premium:

   $$1.71 \times 5.08\% = 8.69\%$$

   Then, simply adding the risk free rate and the risk premium, we get the expected return on Amazon's stocks:

   $$2.41\% + 8.69\% = 11.10\%$$

   Finally, the market expected return is simply the sum of the market risk premium and the risk free rate:

   $$5.08\% + 2.41\% = 7.49\%$$

2. Assume that the expected return on Apple Inc. stocks is currently 7.89%. Yahoo Finance estimates that its stocks beta is 1.08. If the risk free rate is 2.4%, what is the implied market risk premium?

   Remember that the risk premium for an asset is its beta times the market risk premium. Thus, the market risk premium is the ratio of the asset's risk premium to the asset's beta:

   $$\frac{(7.89\% - 2.4\%)}{1.08} = 5.08\%$$

3. Late in 2014, a privately held company's CFO estimates that its shareholder expect a 12.15% return on their shares of stock currently. If the risk free rate was 2.17% at that time, and the market risk premium 5.78%, what is the beta estimate that the CFO used to get the expected return?

   Using equation 3 we can write:

   $$\beta_i = \frac{(E(r_i) - r_f)}{(E(r_M) - r_f)}$$

   Thus we get:

   $$\frac{(12.15\% - 2.17\%)}{5.78\%} = 1.7266$$

4. The Space-Y company is listed on a "small caps" stock market and its stock closed at $143.58 yesterday. There is 1.2 million shares of stocks outstanding, and their beta is estimated to 1.82. The company has no debt so far, but it is considering a $50 millions bonds issue for investment purposes. If the marginal tax rate of Space-Y is 38%, what would happen to its beta?

   As Space-Y does not have debt yet, its current beta is an unlevered beta. What will happen when the company issues debt is that it will add financial risk to its current activity risk, and the beta will increase accordingly.

   To calculate the new beta, we need first to calculate what the company leverage will be:

   $$\frac{50,000,000}{(1,200,000 \times 143.58)} = 0.2902$$

Using equation 6 we can then write:

$$1.82 \times (1 + 0.2902 \times (1 - 0.38)) = 2.1475$$

5. In 2011, Nebula, Inc. is a privately held company. Its CFO whishes to get an estimate of the expected return on its stocks before going public, by using the CAPM. In order to estimate its beta, it found a competitor (thus a company with a similar activity) which beta is 1.32. The competitor has a 38% leverage, but Nebula's leverage is estimated to 0.18 only. Assume that both companies have a marginal tax rate of 30% and use 1.88% and 6% as the risk free rate and the market risk premium, respectively. What is your estimation for the expected return on Nebula's stocks?

   From the competitor's beta and leverage, we can find the unlevered beta, which is the same for Nebula as both companies have the same activity:

$$\beta_{UL} = \frac{1.32}{(1 + 0.38 \times (1 - 0.30))} = 1.0427$$

   We then use the unlevered beta and Nebula's leverage to find Nebula's beta:

$$1.0427 \times (1 + 0.18 \times (1 - 0.30)) = 1.1740$$

   Note that, logically, Nebula's beta is lower than its competitor's one, as Nebula's leverage is lower than the competitor's leverage.

   Now we use Nebula's beta and the CAPM to find the expected return on Nebula's stocks:

$$1.88\% + 1.1740 \times 6\% = 8.92\%$$